Routing in Small-Worlds

ACN Master Project Proposal

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Context

Expressions like *Small-World*[1] or *Six degrees of separation*[2] are quite common in today's culture. One striking feature of small-world networks is the ability to transmit a message to a complete stranger using only a few friend-to-friend transmissions.

In 2000, Kleinberg proposed a theoretical model based on an augmented grid to explain this phenomenum [Kle00]. Starting from a two-dimensional grid, the key idea is to add shortcuts drawn according to some distribution, and to apply a simple greedy algorithm to find short routes.

Recently, the original paper was revisited from a simulation-based perspective [CM17]. Using a new algorithm that can draw an augmenting link in an almost constant time, new results were shown: the augmented scheme proposed by Kleinberg is in practice more robust than predicted by the asymptotic behavior, even for very large finite grids; the performance bounds of Kleinberg's greedy routing algorithm can be tightened; Kleinberg's model gives results in line with real-life experiments (*Six Degrees of Separation* phenomenum).

Objectives

The goal of the project is to provide a soft initiation to research by working on an extension of the results given in [CM17]. There are several options:

[1] `https://en.wikipedia.org/wiki/It%27s_a_Small_World`
[2] `https://en.wikipedia.org/wiki/Six_Degrees_of_Kevin_Bacon`

Giant Fly Swatter The Giant Fly Swatter is a graph structure where the augmenting scheme proposed by Kleinberg theoretically fails if applied directly [DHLS05]. Adapting the simulator from `https://github.com/balouf/Kleinberg`, the goal of the project could be to check how the vanilla augmenting scheme performs in practice on this graph.

Power Law Distributions In [FG14], a theoretical extension is proposed where the number of shortcuts per node follows a power-law distribution. It is proved to result asymptotically in even shorter routes. The goal of the project could be to simulate the model on finite grids and compare the performance with the asymptotic results.

Proving the new bounds Based on simulations results, [CM17] proposes new asymptotic bounds for the performance of greedy routing in Kleinberg's model. These bounds are similar to those proposed for the ring model in [BFKK01]. The goal of the project could be to *prove* these bounds for the grid.

Reasonable exponents For a given, finite, grid size, simulations show that there are multiple augmenting schemes that lead to efficient routing. The goal of the project could be to give theoretical grounds to these observations.

The choice of the subject will be discussed with the supervisors, depending on the interest and background of the student.

Skills

All four choices require to begin with a bibliographical study. The first two proposals are more programming-oriented. The last two proposals are theory-oriented. They require a deep understanding of the proofs from cited articles, and some background in graph theory and probability.

References

[BFKK01] L. Barrière, P. Fraigniaud, E. Kranakis, and D. Krizanc. Efficient routing in networks with long range contacts. In *DISC '01*, pages 270–284, 2001.

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